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ON WEAK POSITIVE SUPERCYCLICITY

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ABSTRACT

A bounded linear operator T on a separable complex Banach space X is called weakly supercyclic if there exists a vector $x \in X$ such that the projective orbit $\{\lambda T^n x : n \in \mathbb{N}\ \lambda \in \mathbb{C}\}$ is weakly dense in X. Among other results, it is proved that an operator T such that $\sigma_p(T^\star) = \emptyset$, is weakly supercyclic if and only if T is positive weakly supercyclic, that is, for every supercyclic vector $x \in X$, only considering the positive projective orbit: $\{rT^n x : n \in \mathbb{N},\ r \in \mathbb{R}_+\}$ we obtain a weakly dense subset in X. As a consequence it is established the existence of non-weakly supercyclic vectors (non-trivial) for positive operators defined on an infinite dimensional separable complex Banach space. The paper is closed with concluding remarks and further directions.

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1. Introduction and main results.

A bounded linear operator T, defined on a separable Banach space X is said to be supercyclic if there exists $x \in X$ (later called supercyclic for T) such that the orbit $\{\lambda T^n x : \lambda \in \mathbb{C},\, n \in \mathbb{N}\}$ is dense in X. On the other hand, an operator T is called cyclic if there exists a vector x such that the set: $\text{span}\{T^n x : n \geq 1\}$ is dense in X. For a survey of results related with Supercyclicity, the reader can see [9] and [13].

Recently, weaker forms of Supercyclicity (nearer to the cyclicity notion) have appeared in the literature ([6, 8]). For instance, we will say that T is weakly supercyclic if for some vector $x \in X$, the projective orbit $\{\lambda T^n x : \lambda \in \mathbb{C},\, n \in \mathbb{N}\}$ is weakly dense in X. We will also say that T is positive weakly supercyclic if for each weakly supercyclic vector x, the subset $\{\lambda T^n x : \lambda \in \mathbb{R}_+,\, n \in \mathbb{N}\}$ is weakly dense in X. The existence of weakly supercyclic operators which are not supercyclic (that is, the projective orbit is not dense in the norm topology) was established in [17].

The interest in the study of different types of operators' orbits, arises from the invariant subspace problem which remains open in case of the separable Hilbert space. For a negative solution to the invariant subspace problem in a separable Banach space, the reader can see [7] and [16]. For a remarkable result in the positive direction, the reader can see [12].

The invariant subspace problem remains open even for positive operators defined in an ordered vector space or even in a Banach lattice. For a good recent survey on this topic the reader can see [1] and for some recent advances in this direction see [2, 3].

Our principal result establishes the following

Theorem 2.1: *Let $\mathcal{M}$ be a semigroup of continuous linear operators acting on a complex topological vector space X. Let us suppose that there is an operator $T \in \mathcal{M}$ satisfying*

(1) *$p(T)$ has dense range for all polynomials p such that $p(T) \neq 0$, and T is not a multiply of the identity.*
(2) *$TS = ST$ for all $S \in \mathcal{M}$.*

If there exists a vector $x \in X$ such that the set

$$\{\lambda S x : |\lambda| = 1,\ S \in \mathcal{M}\}$$

is dense in X, then, the set

$$\{Sx : S \in \mathcal{M}\}$$

is dense in X.

Recall that, a continuous linear operator T, defined on a topological vector space X, is hypercyclic if there exists a vector $x \in X$ such that the orbit $\{T^n x : n \in \mathbb{N}\}$ is dense in X. Theorem 2.1 will be useful to prove also some properties of hypercyclic operators defined on a complex topological vector space X. For instance, as a consequence of Theorem 2.1 we can show that a bounded operator T, on a separable Banach space X is weakly hypercyclic if and only if αT is weakly hypercyclic for every α of modulus 1. This result complements some spectral properties of weakly hypercyclic operators discovered in [15].

We will obtain several consequences from Theorem 2.1. One of the most remarkable is the following contribution to the invariant subspace problem for positive operators

THEOREM 2.6: *Let T be a bounded linear operator defined on a separable complex Banach space X. Let us suppose that T is positive with respect to a cone C. Then, there exists a non-trivial, non weakly supercyclic vector for T.*

The previous result is a direct consequence of a general result on weakly Positive Supercyclicity which complements the Positive Supercyclicity Theorem discovered in [10]. The reader can see in the survey [11] that the Positivity plays a determinant role in the Supercyclicity setting. Section 3 closes the paper with some open problems and further directions.

2. Non weakly supercyclic vectors for positive operators.

The proof of the main result is stated for semigroups of continuous linear operators defined on a complex topological vector space X. We will prove this more general result and we deduce from it several consequences.

THEOREM 2.1: *Let $\mathcal{M}$ be a semigroup of continuous linear operators acting on a complex topological vector space X. Let us suppose that there is an operator $T \in \mathcal{M}$ satisfying*

(1) *$p(T)$ has dense range for all polynomials p such that $p(T) \neq 0$ and T is not a multiply of the identity.*

(2) *$TS = ST$ for all $S \in \mathcal{M}$.*

If there exists a vector $x \in X$ such that the set

$$\{\lambda Sx : |\lambda| = 1,\ S \in \mathcal{M}\}$$

is dense in X, then, the set

$$\{Sx : S \in \mathcal{M}\}$$

is dense in X.

Proof. Let us denote by X_0 the subset of $x \in X$ such that the orbit

$$\{\lambda Sx : |\lambda| = 1,\ S \in \mathcal{M}\}$$

is dense in X.

For $x, y \in X$ put

$$\gimel(x,y) = \left\{\mu \in \mathbb{C} : |\mu| = 1\,,\ \mu y \in \overline{\{Sx : S \in \mathcal{M}\}}\right\}.$$

Firstly, we will see that the set $\gimel(x,y)$ is a closed subset of the unit circle $\mathbb{T} = \{\mu \in \mathbb{C} : |\mu| = 1\}$. Indeed, if $\gimel(x,y) = \emptyset$ then the above assertion is already proved. Let us suppose that $\gimel(x,y) \neq \emptyset$ and let $\{\mu_m\} \subset \gimel(x,y)$ be a sequence converging to $\mu \in \mathbb{T}$. Let V, W neighbourhoods of the origin such that $V + V \subset W$. We claim that $S_m x \in \mu y + W$ for some m. Indeed, since $\mu_m \to \mu$ there exists m_0 such that $\mu_m y \in \mu y + V$ for all $m \geq m_0$. For each $m \geq m_0$, since $\mu_m \in \gimel(x,y)$ there exists $S_m \in \mathcal{M}$ such that $S_m x \in \mu_m y + V$. Therefore, if $m \geq m_0$

$$S_m x \in \mu_m y + V \subset \mu y + V + V \subset \mu y + W.$$

Since W was arbitrary, we obtain that $\mu \in \gimel(x,y)$, which yields the desired result.

Now, let us prove that if $x \in X_0$, then $\gimel(x,y) \neq \emptyset$ for all $y \in X$. We divide the circle $\partial\mathbb{D}$ into two closed arcs F_1 and F_2 on length π. Let us denote by $C_i = \{\lambda Sx : \lambda \in F_i,\ S \in \mathcal{M}\}$, $i = 1, 2$, then

$$\{\lambda Sx : |\lambda| = 1,\ S \in \mathcal{M}\} = C_1 \cup C_2$$

and therefore

$$X = \overline{\{\lambda Sx : |\lambda| = 1,\ S \in \mathcal{M}\}} = \overline{C_1} \cup \overline{C_2}.$$

Now, since $y \in X$ then $y \in \overline{C_1}$ or $y \in \overline{C_2}$. Suppose without loss that $y \in \overline{C_1}$. Again, let us divide F_1 as the union of two closed arcs of length $\pi/2$,

$F_1 = F_1^2 \cup F_2^2$, then we obtain that

$$y \in \overline{\{\lambda Sx : \lambda \in F_1^2,\, S \in \mathcal{M}\}}$$

or

$$y \in \overline{\{\lambda Sx : \lambda \in F_2^2\, S \in \mathcal{M}\}}.$$

By induction, we can construct a sequence or closed arcs F^n, such that $F^n \subset F^{n+1}$, $\mathrm{length}(F^n) = \pi/2^{n-1}$, and such that

$$y \in \overline{\{\lambda Sx : \lambda \in F^n,\, S \in \mathcal{M}\}}.$$

By Cantor's Theorem there exists a unique $\mu \in \bigcap_{n=1}^{\infty} F^n$. We claim that $\mu^{-1} \in \daleth(x,y)$. Indeed, let V, W two neighbourhoods of the origin such that $V + V \subset W$. Since the product by scalars is continuous, there exists $\varepsilon > 0$ such that $\lambda^{-1}y \in \mu^{-1}y + V$ for all λ such that $|\lambda - \mu| \leq \varepsilon$. Let us consider the circumference arc $G_{\mu,\varepsilon} = \{\lambda \in \mathbb{T} : |\lambda - \mu| < \varepsilon\}$, and let us observe that

$$y \in \overline{\{\lambda Sx : \lambda \in G_{\mu,\varepsilon},\, S \in \mathcal{S}\}},$$

this follows because $F^n \subset G_{\mu,\varepsilon}$ for some n. Therefore, there exists $S' \in \mathcal{M}$ and $\lambda \in G_{\mu,\varepsilon}$ such that

$$\lambda S'x \in y + V,$$

which implies that

$$S'x \in \lambda^{-1}y + V \subset \mu^{-1}y + V + V \subset \mu^{-1}y + W.$$

Since W was arbitrary, $\mu^{-1} \in \daleth(x,y)$ therefore, $\daleth(x,y) \neq \emptyset$.

Now, let us prove the following transitivity property. Let $x, y \in X_0$ and $z \in X$ such that $\mu_1 \in \daleth(x,y)$ and $\mu_2 \in \daleth(y,z)$. Then, let us see that $\mu_1\mu_2 \in \daleth(x,z)$.

Indeed, let us fix V an arbitrary neighbourhood of the origin. Since $\mu_2 \in \daleth(y,z)$ there exists $S_1 \in \mathcal{M}$ such that

$$S_1 y \in \mu_2 z + V.$$

Now, since $y \in \overline{\{\mu_1^{-1}Sx : S \in \mathcal{M}\}}$, using the continuity of S_1, there exists $S_2 \in \mathcal{M}$ such that

$$S_1\mu_1^{-1}S_2 x \in \mu_2 z + V$$

which implies that $S_1S_2x \in \mu_1\mu_2 z + V$. That is, $\mu_1\mu_2 \in \daleth(x,z)$.

As a consequence, $\daleth(x,x)$ is a non-empty closed subsemigroup of the unit circle $\mathbb{T}$.

If $\gimel(x,x)=\mathbb{T}$ then by the transitivity property proved above, we have that $\gimel(x,y)=\mathbb{T}$ for all $y\in X$, therefore the set $\{Sx : S\in\mathcal{M}\}$ is dense in X.

In what follows we shall suppose that $\gimel(x,x)\neq\mathbb{T}$ and we will show that this assumption leads to a contradiction. Firstly, we will prove the following claim.

Claim: *Let us suppose that $\gimel(x,x)\neq\mathbb{T}$ then:*

1) *There exists an integer k such that*

$$\gimel(x,x)=\{e^{\frac{2\pi ij}{k}} : j=0,1,\dots,k-1\}.$$

2) *For every $y\in X_0$ there exists $\mu_y\in\mathbb{T}$ such that*

$$\gimel(x,y)=\{\mu_y e^{\frac{2\pi ij}{k}} : j=0,1,\dots,k-1\}.$$

3) *The function f defined on the space $\vee\{x,Tx\}\setminus\{0\}$ by $f(y)=\mu^k$ where μ is any element in $\gimel(x,y)$ is well defined and continuous.*

Proof of the claim. To prove 1), let us consider the number

$$s=\inf\{t>0 : e^{2\pi it}\in\gimel(x,x)\}.$$

Let us see that $s>0$. Otherwise, if $s=0$ then there exists a sequence $t_n>0$ such that $t_n\to 0$ and $e^{2\pi it_n}\in\gimel(x,x)$. But the last fact implies that $\gimel(x,x)$ is dense in $\mathbb{T}$. Indeed, if for some n, t_n is irrational then since $\gimel(x,x)$ is a subsemigroup then $\gimel(x,x)$ contains a dense subset. Therefore, we can suppose that t_n is rational for all n, but this fact implies that $\gimel(x,x)$ contains the n-roots of the unity for all n, which is a dense subset in $\mathbb{T}$. Therefore, $e^{2\pi is}\in\gimel(x,x)$ and $s>0$ is a rational number, hence $1\in\gimel(x,x)$.

Let $k=\min\{n\in\mathbb{N} : ns\geq 1\}$. If $ks>1$, since $\gimel(x,x)$ is a semigroup $e^{2\pi i(ks-1)}\in\gimel(x,x)$, but $0<ks-1<s$ which is a contradiction to the definition of s. Then, $ks=1$ and $\gimel(x,x)\supset\{e^{2\pi ij/k} : j=0,1,\dots,k-1\}$.

Let us suppose that $\mu\in\gimel(x,x)\setminus\{e^{2\pi ij/k} : j=0,1,\dots,k-1\}$. Then $\mu=e^{2\pi it}$ with $t\in(0,1)$ and there exists $j_0\in\{0,1,\dots,k-1\}$ such that $j_0/k<t<(j_0+1)/k$. But this implies that $e^{2\pi i(t-j_0/k)}\in\gimel(x,x)$ with $0<j_0/k-t<s$, but this is a contradiction with the definition of s. Therefore we have proved 1).

Let us prove 2). Since $\gimel(x,y)$ and $\gimel(y,x)$ are non-empty, let $\mu_y\in\gimel(x,y)$ and $\alpha\in\gimel(y,x)$. Let us observe that $\mu_y\gimel(x,x)\subset\gimel(x,y)$. On the other hand, $\gimel(x,y)\alpha=\alpha\gimel(x,y)\subset\gimel(x,x)$ (by the transitivity property), therefore $\gimel(x,x)$

and $\beth(x,y)$ have the same cardinality. Therefore,

$$\beth(x,y) = \{\mu_y e^{2\pi i j/k} : j = 0,1,\ldots,k-1\}$$

which yields the desired equality.

Finally, to prove 3), let us observe that by 2) the function f is well-defined. Moreover, f is continuous: Suppose on the contrary that there are non-zero vectors $u_n, u \in \vee\{x, Tx\} \setminus \{0\}$ such that $u_n \to u$ but $f(u_n) \nrightarrow f(u)$ (let us observe that we consider on $\vee\{x, Tx\} \setminus \{0\}$ some metric topology). We can suppose, without loss of generality, that the sequence $f(u_n)$ converges to some $\alpha \in \mathbb{T}$, and $\alpha \neq f(u)$. Let $\mu_n \in \beth(x, u_n)$, passing to a subsequence if it is necessary, we can suppose that μ_n converges to some $\mu \in \mathbb{T}$. We claim that $\mu \in \beth(x,u)$.

Let V, W neighbourhoods of the origin satisfying $V + V \subset W$. Since $\mu_n u_n \to \mu u$, there exists n_0 such that $\mu_n u_n \in \mu u + V$ for all $n \geq n_0$. On the other hand, since $\mu_n \in \beth(x, u_n)$, for each $n \geq n_0$ there exists $S_n \in \mathcal{M}$ such that $S_n x \in \mu_n u_n + V$ which implies that

$$S_n x \in \mu_n u_n + V \subset \mu u + V + V \subset \mu u + W,$$

for all $n \geq n_0$, therefore $\mu \in \beth(x,u)$. Thus, $f(u) = \mu^k$, but

$$\alpha = \lim_{n\to\infty} f(u_n) = \lim_{n\to\infty} \mu_n^k = \mu^k = f(u),$$

a contradiction. ∎

The vectors x and Tx are linearly independent. In other case, there exists $\alpha \in \mathbb{C}$ such that $Tx = \alpha x$. Since $S \in \mathcal{M}$ commutes with T, $S(\mathrm{Ker}(T-\alpha)) \subset \mathrm{Ker}(T-\alpha)$ for all $S \in \mathcal{M}$, and therefore $\{\mu S x : \mu \in \mathbb{T}, S \in \mathcal{M}\}^- = X \subset \mathrm{Ker}(T-\alpha)$, which is a contradiction because by hypothesis T cannot be a multiply of the identity. Therefore, $zx + (1-|z|)Tx \in X_0$ for all $z \in \mathbb{C}$, indeed since $D = \{\lambda S(x) : |\lambda| = 1, S \in \mathcal{M}\}$ is dense and T satisfies 2)

$$\{\lambda S(zx + (1-|z|)Tx) : |\lambda| = 1, S \in \mathcal{M}\} = (zI + (1-|z|)T)(D)$$

which is dense because $zI + (1-|z|)T$ has dense range.

Let us denote by $\mathbb{D}$ the closed unit disk, and let us define the function $g : \mathbb{D} \to \mathbb{T}$ by $g(z) = f(zx + (1-|z|)Tx)$. For all z in the boundary of $\mathbb{D}$, we have that $\beth(x, zx) = z^{-1}\beth(x,x)$. Indeed, let $\mu \in \beth(x, zx)$, that is, there exists a subsequence $\{S_n\} \subset \mathcal{M}$ such that $S_n x \to \mu z x$ as $n \to \infty$, but this implies that $\mu z \in \beth(x,x)$ and this proves the assertion.

Let us observe the function g on $\mathbb{T}$, $g(z) = f(zx) = z^{-k}f(x) = z^{-k}$. The function g provides a homotopy between the constant path $\gamma_1(t) = 0$ and the path $\gamma_2(t) = g(e^{it}) = e^{-kit}$ which has the winding number $-k$, a contradiction. ∎

Next, we will obtain some consequences from Theorem 2.1.

THEOREM 2.2: *Let T be a hypercyclic operator defined on a complex topological vector space X. Then $x \in X$ is hypercyclic for T if and only if x is hypercyclic for λT for all $\lambda \in \mathbb{T}$.*

Proof. Let us consider the semigroup $\mathcal{M} = \{\lambda^n T^n : n \in \mathbb{N}\}$. Since T is hypercyclic, for each hypercyclic vector for T, $x \in X$, we have that

$$\{\mu\lambda^n T^n x : n \in \mathbb{N},\, \mu \in \mathbb{T}\}$$

is dense in X. Since the orbit of T is dense (in particular its closure has nonempty interior), by the results in [18], we have that $p(T)$ has dense range for every polynomial p such that $p(T) \neq 0$. Hence, the hypothesis of Theorem 2.1 are fulfilled and we obtain that T and λT have the same set of hypercyclic vectors. ∎

As a consequence we obtain the following corollary which complements the spectral properties of weakly hypercyclic operators discovered in [15].

COROLLARY 2.3: *Let T be a weakly hypercyclic operator defined on a complex Banach space X. Then $x \in X$ is weakly hypercyclic for T if and only if x is weakly hypercyclic for λT for all $\lambda \in \mathbb{T}$.*

Now, we obtain some consequences for supercyclic operators.

THEOREM 2.4: *Let X be a complex locally convex vector space, and let T be a bounded linear operator on X. Let us suppose that $\sigma_p(T^\star) = \emptyset$. Then, the vector x is supercyclic for T if and only if the set*

$$\{rT^n x : n \in \mathbb{N},\, r \in \mathbb{R}_+\}$$

is dense in X.

Proof. Since $\sigma_p(T^\star) = \emptyset$ a direct consequence of the Hahn–Banach Theorem asserts that $p(T)$ has dense range whenever $p(T) \neq 0$. The proof follows directly applying Theorem 2.1 to the following semigroup: $\{rT^n : n \in \mathbb{N},\, r \in \mathbb{R}_+\}$. ∎

In particular, for the weak topology in a Banach space we have

Corollary 2.5: *Let T be a weakly supercyclic operator on a complex Banach space X. Let us suppose that $\sigma_p(T^\star) = \emptyset$. Then, the vector x is weakly supercyclic for T if and only if the subset*

$$\{rT^n x : n \in \mathbb{N},\, r \in \mathbb{R}_+\}$$

is weakly dense in X.

And finally, as a consequence

Theorem 2.6: *Let X be a complex separable Banach space, and let T be a bounded linear operator on X. Let $C \subset X$ be a closed cone, and let us suppose that $T(C) \subset C$, then there exists a non trivial, weakly supercyclic vector for T.*

Proof. If $\sigma_p(T^\star) \neq \emptyset$ then, it is well-known that T has a non-trivial hyperinvariant subspace (see [1, Theorem 3.4]). Therefore, a non-trivial, non-cyclic vector for T, which, in particular, is a non-weakly supercyclic vector. We can suppose without loss that $\sigma_p(T^\star) = \emptyset$. In that case, let us observe that for every element $x \in C \setminus \{0\}$ we have that the set

$$\{rT^n x : r \in \mathbb{R}_+, n \in \mathbb{N}\} \subset C$$

and therefore cannot be weakly dense. Therefore, using Corollary 2.5, we have that x is a non-trivial non weakly supercyclic vector for T. ∎

Remark 2.7: Using the results obtained in [14] and Corollary 2.5 it is possible to obtain in the Hilbert setting (even for reflexive Banach spaces), non weakly supercyclic vectors (non-trivial) for power bounded operators whose spectral radius is equal to 1. This result is related to the famous result obtained by Brown–Chevreau and Pearcy [5].

3. Concluding remarks.

It would be interesting to extend these results to weaker forms of non-cyclicity. The following notion was pointed out by P. Enflo.

> A bounded linear operator T defined on a separable Banach space X is cyclic with support $N = 2$ if there exists a vector

$x \in X$ such that the set

$$\{aT^n x + bT^m x : a, b \in \mathbb{C},\ m, n \in \mathbb{N}\}$$

is dense in X.

Firstly, it is not known about the existence of cyclic operators with arbitrary support $N \geq 2$, which are not supercyclic. On the other hand, it will be interesting to know if for every cyclic vector x with finite support, for instance $N = 2$, the subset

$$\{rT^n x + sT^m x : r, s \in \mathbb{R}_+,\ m, n \in \mathbb{N}\}$$

remains dense in X.

Recently, it has been introduced the notion of N-supercyclicity (see [8]). An operator T is said to be $N = 2$-supercyclic if there exist two linearly independent vectors $u, v \in X$ such that the set

$$\{aT^n u + bT^n v : a, b \in \mathbb{C},\ n \in \mathbb{N}\}$$

is dense in X. This is another generalization of the notion of Supercyclicity (see [4] for more properties of N-supercyclic operators).

It will be interesting to obtain a "Positive version" of Corollary 2.4 for N-supercyclic operators.

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